METHOD OF CALCULATING THE FLOW OF A VAPOR-DROP MIXTURE ABOUT BODIES IN THE PRESENCE OF FRAGMENTATION AND VAPORIZATION EFFECTS

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In the flow of gas-drop mixtures (mixtures of a gas with a large number of liquid drops) about bodies, the drops are deformed and fragmented due to the difference in the velocities of the phases. The fine secondary drops formed due to the fragmentation of the coarse initial drops are rapidly vaporized and, as a result, have a greater effect on the character of flow about the body than do solid particles. The present study is devoted to calculation of the flow of vapor-drop mixtures about bodies. The problem is formulated, the basic equations are written, and features of the numerical algorithm are discussed. Some of the calculated results are reported. The effect of inert solid particles suspended in a gas on conditions of flow about bodies was studied in [1-3].

1. Basic Assumptions. We will suppose that the mixture is monodisperse, the drops are incompressible and do not collide with one another, the effects of viscosity and heat conduction are significant only in processes involving interaction of the phases, drop fragmentation occurs under certain conditions and takes place by the mechanism of stripping of the surface layer, the velocities and temperatures of the already-separated fine drops and the vapor are the same, and some of the separated drops are vaporized in the gas flow. Here, the vaporization occurs under equilibrium conditions (the temperatures of the fine drops and the vapor are equal to the saturation temperature). We also assume that phase transformations can take place on the surface of the coarse drops only in the case where stripping of the surface layer does not occur. Below, the index 1 is used to denote parameters of the medium consisting of a one-velocity, one-temperature mixture of vapor and small drops (the "effective vapor"), the indices 1v and 12, respectively, denote parameters of the mixture's components – the vapor and small drops – and the index 2 denotes parameters of the coarse drops.

The following relations link the true densities ρ_1^0, ρ_{1v}^0, ρ_2^0, the corrected (spread over the volume of the mixture) densities ρ_1, ρ_{1v}, ρ_{12}, ρ_2, and the volumetric contents α_1, α_{1v}, α_{12}, α_2 of the effective vapor, its vapor and liquid components, and the coarse drops

$$\begin{gathered}\rho_1 = \alpha_1\rho_1^0,\ \rho_2 = \alpha_2\rho_2^0,\ \alpha_1 + \alpha_2 = 1,\ \alpha_2 = n\pi d^3/6,\\ \rho_{1v} = \alpha_{1v}\rho_{1v}^0,\ \rho_{12} = \alpha_{12}\rho_2^0,\ \rho_{1v} + \rho_{12} = \rho_1,\ \alpha_{1v} + \alpha_{12} = \alpha_1\end{gathered} \tag{1.1}$$

(n and d are the number of coarse drops per unit volume of the mixture and their diameter).

2. Differential Equations of Motion. Within the framework of the above assumptions, the equations of mass conservation for the effective vapor and coarse drops and the equation of conservation of the number of coarse drops for two-dimensional nonsteady motion with planar symmetry have the form

$$\begin{gathered}\frac{\partial\rho_2}{\partial t} + \frac{\partial\rho_1 u_1}{\partial x} + \frac{\partial\rho_1 v_1}{\partial y} = nj_2^s + nj_2^v,\\ \frac{\partial\rho_2}{\partial t} + \frac{\partial\rho_2 u_2}{\partial x} + \frac{\partial\rho_2 v_2}{\partial y} = -nj_2^s - nj_2^v,\ \frac{\partial n}{\partial t} + \frac{\partial nu_2}{\partial x} + \frac{\partial nv_2}{\partial y} = 0\end{gathered} \tag{2.1}$$

(u_i and v_i are components of the velocity vector $\mathbf{v}_i$ (i = 1, 2) of the effective vapor and the coarse drops over the axes x and y; j_2^v and j_2^s are the rates of vaporization and stripping of one coarse drop).

We write the momentum conservation equations for the effective vapor and the coarse drops in the form

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$$\frac{\partial\rho_1\mathbf{v}_1}{\partial t}+\frac{\partial\rho_1 u_1\mathbf{v}_1}{\partial x}+\frac{\partial\rho_1 v_1\mathbf{v}_1}{\partial y}=-\nabla p-n\mathbf{f}+nj_2^s\mathbf{v}_2+nj_2^v\mathbf{v}_2,$$
$$\frac{\partial\rho_2\mathbf{v}_2}{\partial t}+\frac{\partial\rho_2 u_2\mathbf{v}_2}{\partial x}+\frac{\partial\rho_2 v_2\mathbf{v}_2}{\partial y}=n\mathbf{f}-nj_2^s\mathbf{v}_2-nj_2^v\mathbf{v}_2. \qquad (2.2)$$

Here, p is pressure; $\mathbf{f}$ is the force acting from the direction of the carrier phase on a single coarse drop. It is assumed that the characteristic velocity of the liquid undergoing stripping and vaporization (from the surface of the coarse drops) is equal to the velocity of the coarse drops.

The equation describing the flow of heat to the drops and the equation for the total energy of the mixture as a whole are written in the form

$$\frac{\partial\rho_2 e_2}{\partial t}+\frac{\partial\rho_2 e_2 u_2}{\partial x}+\frac{\partial\rho_2 e_2 v_2}{\partial y}=-nq_{2\Sigma}-nj_2^s e_2-nj_2^v e_{2s},$$
$$\frac{\partial(\rho_1E_1+\rho_2E_2)}{\partial t}+\operatorname{div}(\rho_1E_1\mathbf{v}_1+\rho_2E_2\mathbf{v}_2)+\operatorname{div}(\alpha_1p\mathbf{v}_1+\alpha_2p\mathbf{v}_2) \qquad (2.3)$$

where e_1, e_2, E_1, and E_2 are the internal and total energies of the effective vapor and the coarse drops; $q_{2\Sigma}$ is the rate of flow of heat from the interior of the coarse drop to its surface (the temperature of the bulk of the liquid may differ from the temperature on its surface); e_{2s} is the internal energy of the coarse drops in the saturated state.

Using the kinetic energy theorem, it is easy to use Eqs. (2.1)-(2.3) to obtain an equation describing the flow of heat to the effective vapor

$$\frac{\partial\rho_1e_1}{\partial t}+\operatorname{div}(\rho_1e_1\mathbf{v}_1)=\frac{\alpha_1p}{\rho_1^0}\frac{d_1\rho_1^0}{dt}+n\mathbf{f}(\mathbf{v}_1-\mathbf{v}_2)-nq_{1\Sigma}+\left(nj_2^s+nj_2^v\right)(\mathbf{v}_1-\mathbf{v}_2)^2/2+nj_2^se_2+nj_2^v\left(i_{1s}-p/\rho_1^0\right),$$
$$\frac{d_1\rho_1^0}{dt}=\frac{\partial\rho_1^0}{\partial t}+u_1\frac{\partial\rho_1^0}{\partial x}+v_1\frac{\partial\rho_1^0}{\partial y}. \qquad (2.4)$$

Here, $q_{1\Sigma}$ is the rate of heat flow from the effective vapor to the surface of the coarse drop; i_{1s} is the enthalpy of the effective vapor in the saturated state. The first term in the right side of (2.4) is connected with the change in the internal energy of the effective vapor due to its compression (or expansion), the second term is the work done by the phase interaction force, the third term is the rate of heat flow from the effective vapor to the phase boundary, and the remaining terms are connected with the change in the internal energy of the effective vapor due to the vaporization of coarse drops and stripping of their surface layer. In contrast to (2.3), Eq. (2.4) has a nondivergent and appreciably more cumbersome form.

3. Equations of State of the Phases. We will assume that the vapor is calorically perfect, while the condensed phase will be regarded as an incompressible medium with a constant heat capacity. Then the equations of state of the effective vapor and the coarse drops can be written in the form

$$p=\rho_1R_1T_1,\ e_1=c_{V1}T_1,$$
$$R_1=x_{1v}R_v/\alpha_{1v},\ c_{V1}=x_{1v}c_{Vv}+x_{12}c_2, \qquad (3.1)$$

$$x_{1v}+x_{12}=1,\ x_{1v}=\rho_{1v}/\rho_1,\ x_{12}=\rho_{12}/\rho_1;$$
$$\rho_2^0=\text{const},\ e_2=c_2T_2\qquad(R_v,\ c_{Vv},\ c_2=\text{const}), \qquad (3.2)$$

where R_v, c_{Vv}, and c_2 are the gas constant and the heat capacity of the gas and liquid at constant volume; T_1 and T_2 are the temperatures of the gas and drop; x_{1v} and x_{12} are the relative mass fractions of vapor and fine drops in the effective vapor. Due to the variability of R_1, the effective vapor is not calorically perfect.

Since the mass fractions of the vapor x_{1v} and the fine drops x_{12} depend on the rate of phase transformations taking place on the fine drops, then system (2.1)-(2.4), (3.1), and (3.2) is still not closed. To determine x_{1v} and x_{12}, we use the assumption of phase equilibrium in the effective vapor. Here, it is supposed that the temperature of the effective vapor in the presence of fine drops is equal to the saturation temperature:

$$T_1 = T_s(p). \tag{3.3}$$

Here, T_s is the saturation temperature, connected with the pressure by the Clapeyron–Clausius equation. Inserting (3.3) into (3.1) and taking (1.1) into account, we find equations which link the unknown pressure p and the mass and volume contents x_{1v}, α_{1v}:

$$\begin{gathered} p = \rho_1 R_1 T_s(p),\ e_1 = c_{V1} T_s(p),\ R_1 = x_{1v} R_v/\alpha_{1v}, \\ c_{V1} = x_{1v} c_{Vv} + (1 - x_{1v})\, c_2,\ \alpha_{1v} = 1 - [\rho_2 + (1 - x_{1v})\, \rho_1]/\rho_2^0. \end{gathered} \tag{3.4}$$

With known values of ρ_1 and ρ_2 and the internal energy e_1, obtained during integration of differential equations (2.1)-(2.3), and with an assigned relation $T_s(p)$, Eqs. (3.4) can be used to determine three unknown parameters of the effective vapor: p, x_{1v}, and α_{1v}.

We can use (3.4) to obtain the algebraic equation

$$p - \frac{[A(e_1) - T_s(p)]\, T_s(p)}{B(\rho_1, \rho_2)\, T_s(p) + D\,[A(e_1) - T_s(p)]} = 0, \tag{3.5}$$

where $A(e_1) = \frac{e_1}{c_2}$; $B(\rho_1, \rho_2) = \left(1 - \frac{\rho_1 - \rho_2}{\rho_2^0}\right) \frac{1 - c_2/c_{Vv}}{(\gamma_v - 1)\, \rho_1 c_2}$; $D = \frac{1}{\rho_2^0 R_v}$.

The dependence of T_s on pressure is approximated by the following function for a broad range of substances

$$T_s(p) = T_*/(p_* - \ln p) \tag{3.6}$$

(T_* and p_* are constants).

Equation (3.5) cannot normally be solved for p. However, it can readily be solved by iterative procedures. If the pressure is found by means of (3.5), then the mass fraction of the vapor can be determined from the equation

$$x_{1v} = [A(e_1) - T_s(p)]/F\,T_s(p)\ (F = c_{Vv}/c_2 - 1), \tag{3.7}$$

being a consequence of the equation of state $e_1 = c_{V1} T_s$. If here we obtain $x_{1v} \geq 1$, this means that the fine drops are completely vaporized and, instead of the effective vapor, we have normal superheated vapor. In this case, the temperature and pressure of the vapor can be found from (3.1) with $x_{1v} = 1$.

4. Laws of Phase Interaction. To close system (2.1)-(2.3), (3.1)-(3.3), we need to assign laws of phase interaction j_2^s, j_2^v, f , $q_{1\Sigma}$ and $q_{2\Sigma}$. Following [4], the rate of stripping j_2^s of the coarse drops will be assigned as follows:

$$\begin{gathered} j_2^s = \begin{cases} 0, & \mathrm{We}_{12} < \mathrm{We}_c, \\ j_{2*}^s, & \mathrm{We}_{12} \geqslant \mathrm{We}_c, \quad \mathrm{We}_c = 0.5\, \mathrm{Re}_{12}^{0.5}, \end{cases} \\ j_{2*}^s = k\,(\rho_{1v}^0)^{1/6}\,(d/2)^{3/2}\,|\mathbf{v}_1 - \mathbf{v}_2|^{1/2},\ k \cong 1.3 - 2\ \mathrm{kg}^{5/6}/(\mathrm{m}^{3/2}\cdot\mathrm{sec}^{1/2}), \\ \mathrm{We}_{12} = \rho_{1v}^0 d\,|\mathbf{v}_1 - \mathbf{v}_2|^2/\sigma, \quad \mathrm{Re}_{12} = \rho_{1v}^0 d\,|\mathbf{v}_1 - \mathbf{v}_2|/\mu_{1v}. \end{gathered} \tag{4.1}$$

Here, Re_{12}, We_{12}, We_c are the Reynolds number, Weber number, and critical Weber number; σ and μ_{1v} are the surface tension of the drop and the absolute viscosity of the gas.

To assign the rate of vaporization of the drop j_2^v, we use the formula [4, 5]

$$j_2^v = \begin{cases} 0, & j_2^s \neq 0, \\ j_{2*}^v, & j_2^s = 0, \quad j_{2*}^v = (q_{1\Sigma} + q_{2\Sigma})/l \end{cases} \tag{4.2}$$

(ℓ is the heat of vaporization). Equation (4.2) was written with allowance for the assumption that no phase transformation takes place on the coarse drops in the presence of stripping of their surface layer.

Following [4, 5], we will proceed as below in assigning the mechanical interaction f between the gas and the drops and the rate of heat flow to the surface from the carrier phase $q_{1\Sigma}$ and from inside the drop $q_{2\Sigma}$

$$\mathbf{f} = (\pi d^2/8)\,\rho_{1v}^0 C_d\,|\mathbf{v}_1 - \mathbf{v}_2|\,(\mathbf{v}_1 - \mathbf{v}_2),$$

$$q_{1\Sigma} = \begin{cases} 0, & j_2^s \neq 0, \\ \overset{*}{q}_{1\Sigma}, & j_2^s = 0, \end{cases} \quad q_{2\Sigma} = \begin{cases} 0, & j_2^s \neq 0, \\ \overset{*}{q}_{2\Sigma}, & j_2^s = 0, \end{cases} \tag{4.3}$$

$$q_{1\Sigma} = \pi d \lambda_{1v}\,\mathrm{Nu}_1\,(T_1 - T_\Sigma), \quad q_{2\Sigma} = \pi d \lambda_2\,\mathrm{Nu}_2\,(T_2 - T_\Sigma),$$

where C_d is the drag coefficient of the drop; λ_{1V} and λ_2 are the heat-transfer coefficients of the gas and drop; Nu_1 and Nu_2 are the external and internal Nusselt numbers of the drop; T_Σ is the temperature of the drop surface, which is usually assumed to be equal to T_s [5]. In assigning $q_{1\Sigma}$ and $q_{2\Sigma}$, we assumed that under surface-stripping conditions, we could ignore heat transfer between the gas and the surface layer and between the surface layer and the bulk of the drop [4].

To assign C_d and Nu_1, we use the relations [5, 6]

$$C_d = C_d^0\left[1 + \exp\left(-0.423/\mathrm{M}_{12}^{4.63}\right)\right],$$

$$C_d^0 = \begin{cases} 27\,\mathrm{Re}_{12}^{-0.84}, & \mathrm{Re}_{12} < 80, \\ 0.27\,\mathrm{Re}_{12}^{0.217}, & 80 \leqslant \mathrm{Re}_{12} < 10^4, \\ 2, & 10^4 \leqslant \mathrm{Re}_{12}, \end{cases} \tag{4.4}$$

$$\mathrm{Nu}_1 = 2 + 0.6\,\mathrm{Re}_{12}^{1/2}\mathrm{Pr}_1^{1/3}, \quad \mathrm{Pr}_1 = c_{pv}\mu_{1v}/\lambda_{1v}, \quad \mathrm{M}_{12} = |\mathbf{v}_1 - \mathbf{v}_2|/a_{1v}.$$

Here, Pr_1 and M_{12} are the Prandtl and Mach numbers; c_{pV} is the isobaric heat capacity of the gas; a_{1V} is the local speed of sound in the gas. The internal Nusselt number is assumed to be constant [5]: $Nu_2 = 10$.

5. Formulation of the Problem. We examined the external transverse flow of a gas-drop mixture about a flat plate. The theoretical region was assigned in the form of a rectangle. The boundary conditions chosen for the body in the flow were impermeability with regard to the gas phase and disappearance from the flow with regard to the drop. The condition in the incoming flow was used on the left boundary, the symmetry condition was used on the lower boundary, and the condition of continuity of the flow was used on the remaining boundaries. An undisturbed flow existed at the initial moment of time. It was assumed that the mixture was in thermodynamic equilibrium in the incoming flow ($\mathbf{v}_{1\infty} = \mathbf{v}_{2\infty}$, $T_{1\infty} = T_{2\infty}$).

As the characteristic length of the problem L_*, we used the minimum characteristic length $L_* = \min(L_V, L_T, L_m, h)$, where L_V and L_T are relaxation lengths of phase velocity and temperature, L_m is the characteristic stripping length (the distance over which the mass of the drop is substantially reduced due to stripping), and h is the height of the plate. As the characteristic time, it was convenient to take $\tau_* = L_*/|\mathbf{v}_{1\infty}|$. The characteristic lengths L_V, L_T, and L_m were found from the expressions [4, 7]

$$L_v = \frac{2.6\,\rho_2^0 d_\infty}{\rho_{1\infty}^0}, \quad L_T = \frac{\rho_2^0 d_\infty^2 c_2 a_{1\infty} \mathrm{M}_\infty^{0.5}}{5.4\lambda_{1v}\mathrm{Pr}_1^{0.33}\mathrm{Re}_{\infty s}^{0.5}}, \quad L_m = \frac{\rho_2^0 d_\infty^{3/2}\,|\mathbf{v}_{1\infty}|^{1/2}}{k\left(\rho_{1\infty}^0\right)^{1/6}},$$

$$\mathrm{M}_\infty = |\mathbf{v}_{1\infty}|/a_{1\infty}, \quad \mathrm{Re}_{\infty s} = \rho_{1\infty}^0 d_\infty a_{1\infty}/\mu_{1v}$$

(the subscript ∞ pertains to parameters in the incoming flow, M_∞ and $Re_{\infty s}$ are the characteristic Mach and Reynolds numbers). In the investigated ranges of values of the governing parameters, the height of the plate h is always less than the other characteristic lengths. Thus, we took the height of the plate h as the quantity L_*, while as τ_* we took the characteristic time $\tau_* = h/|\mathbf{v}_{1\infty}|$.

6. Analysis of Similarity Conditions. After reducing system (2.1)-(2.3), (3.1)-(3.3), Eqs. (4.1)-(4.4), and the boundary conditions to dimensionless form, we find that the similarity criteria of the process of flow about the plate take the form of twelve dimensionless quantities: γ_v, M_∞, m, $\alpha_{1\infty}$, C, Λ, Pr_1, $\mathrm{Re}_{\infty s}$, $\bar{L}_v$, $\bar{L}_m$, $\bar{l}_\infty$, $\mathrm{We}_{\infty s}$ ($m = \rho_{2\infty}/\rho_{1\infty}$, $C = c_{Vv}/c_2$, $\Lambda = \lambda_{1v}/\lambda_2$, $\bar{L}_v = L_v/h$, $\bar{L}_m = L_m/h$, $\bar{l}_\infty = l_\infty/a_{1\infty}^2$, $\mathrm{We}_{\infty s} = \rho_{1\infty}^0 d_\infty a_{1\infty}^2/\sigma$). Here, the criterion $\alpha_{1\infty}$ is immaterial, since we are studying flows of fairly dilute suspensions ($\alpha_{2\infty} \ll 1$, $\alpha_{1\infty} \sim 1$). To analyze the similarity conditions of the flows, we will separately examine the following three cases.

A. Drop fragmentation and phase transformations in the flow are absent, i.e., j_2^s, $j_2^V = 0$. Here. we have eight basic similarity criteria: γ_V, M_∞, m, C, Λ, Pr_1, $Re_{\infty s}$, $\bar{L}_V$. The constancy of $Re_{\infty s}$, C, Λ, and Pr_1 is necessary to ensure similarity of the flows in the presence of heat transfer between the phases. Interphase friction phenomena usually exert the predominant effect on flow conditions in a gas suspension in the shock layer ahead of the body. Thus, the following parameters can be considered the main criteria of approximate similarity for flows about bodies: γ_V, M_∞, m, $\bar{L}_V$.

B. Drop fragmentation is absent ($j_2{}^s = 0$), but phase transformations do take place on the phase boundary ($j_2{}^v \neq 0$). Then the dimensionless heat of vaporization $\bar{\ell}_\infty$ becomes one of the similarity criteria. Thus, the similarity of flows in the presence of phase transformations is determined by nine criteria: γ_V, M_∞, m, C, Λ, Pr_1, $Re_{\infty s}$, $\bar{L}_V$, $\bar{\ell}_\infty$.

C. Drop fragmentation is present ($j_2{}^s \neq 0$). In this case, the main similarity criteria for the flow of a gas-drop mixture about a plate will be the seven dimensionless parameters: γ_V, M_∞, m, $Re_{\infty s}$, $\bar{L}_m$, $\bar{\ell}_\infty$, $We_{\infty s}$.

7. Analysis of the Calculation Results. We solved the problem of the one-component transverse flow of a vapor-drop mixture about a flat plate. System (2.1)-(2.3), (3.1)-(3.3), with closing relations (4.1)-(4.4) and the assigned initial and boundary conditions, were reduced to dimensionless form and integrated numerically by the coarse particle method [8, 9]. To check the correctness of the program that was written and the accuracy of the results we obtained, we also solved the problem of the transverse flow of a pure gas about the plate. The results of the calculations along the plane of symmetry were compared with results which follow from familiar analytical formulas of gasdynamics (the Rankin–Hugoniot relations for a shock wave and the Bernoulli integral). The comparison showed good agreement between the calculated data and analytical formulas throughout the investigated range of Mach numbers. We also solved the problem of the transverse flow of a gas with solid particles about a flat plate. The numerical solution was compared with the results from [1], where a similar problem was solved. The comparison showed that the calculated results agreed almost completely with the data in [1].

We should note one special feature of the computation which is due to the change in drop diameter as a result of the surface-stripping. Drop diameter may decrease substantially during the calculation as a result. In order to avoid having to make corresponding changes in the space and time steps while maintaining the necessary accuracy, it was proposed that drop diameter not become less than a certain minimum value d_*. If the diameter reached this value, then it was assumed that subsequent stripping of the surface layer of the drop occurred as though it were the result of a reduction in the number of drops while drop diameter remained constant $\bar{d} = d/d_\infty = \bar{d}_* = \text{const}$. In the calculations, $\bar{d}_* = 0.1\text{-}0.2$.

To check for the presence of fine drops in the effective vapor (an equilibrium mixture of vapor with fine drops), the temperature T_1 in each cell of the theoretical region on each time level was compared with $T_s(p)$ [T_1 and T_s were found with the condition of complete vaporization of the fine drops; meanwhile, as the relation $T_s(p)$, we used Eq. (3.6)]. If $T_1 > T_s$, then this unambiguously signified that superheated vapor was present without fine drops and that the values of p and T_1, T_s were found correctly. If $T_1 \leq T_s$, this indicated that the effective vapor contained fine drops. In this case, the phase composition of the effective vapor was found from (3.7). We used simple iteration to find the pressure from Eq. (3.5). We then used the pressure value obtained and (3.7) to determine the mass fraction of vapor x_{1V}. As the initial value of pressure in the iteration formula, we used the pressure on the preceding time level.

We studied the effect of three governing parameters (the Mach number of the incoming flow M_∞, the relative mass content $m = \rho_{2\infty}/\rho_{1\infty}$ and diameter d_∞ of the drops) on the character of flow about the plate. It was assumed that the incoming flow was an equilibrium mixture of vapor with water drops at a pressure of 0.1 MPa (T_∞ = 373 K). The calculations were performed in the following ranges of the dimensionless parameters: $M_\infty = 0.7\text{-}3$; $m = 0.5\text{-}2$. The diameter d_∞ was varied from 200 to 600 μm (here, the values of the characteristic dimensionless parameters $\bar{L}_m$ changed from 1 to 4.8). The results were compared with calculations performed in accordance with frozen and equilibrium schemes corresponding to the limiting situations $d_\infty = \infty$ and $d_\infty = 0$.

Throughout the indicated ranges of the governing parameters, the temperature of the vapor was greater than the saturation temperature over the entire flow field behind the front of the departing shock wave. In connection with this, the fine drops formed during stripping of the coarse drops were completely vaporized in the gas flow. Thus, the calculations were simplified to a certain extent. They showed that the vaporization of separated fine drops

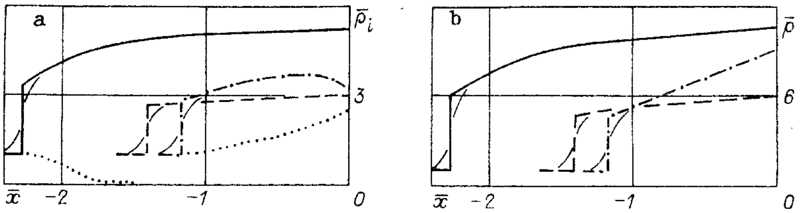

Fig. 1

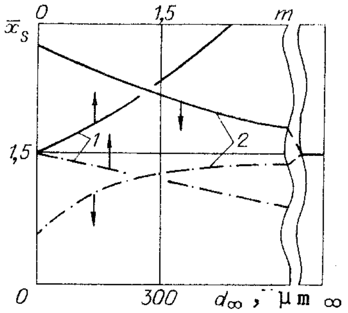

Fig. 2

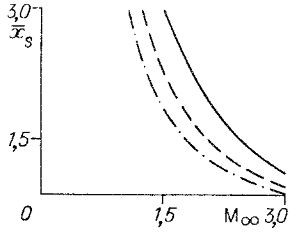

Fig. 3

has a major effect on the process of flow about the plate. Phase transformations on the coarse drops (with the assumption that stripping is absent) have little effect, since there is not sufficient time for such transformations during the characteristic period during which the shock wave passes the coarse drops.

Some of the calculated results are shown in Figs. 1-3. Figure 1 illustrates the effect of the presence of stripping and vaporization of drops on the behavior of the corrected densities of the phases (*a*) $\bar{\rho}_i = \rho_i/\rho_{i\infty}$ ($i = 1, 2$) and $\bar{p} = p/p_\infty$ (b) in a shock wave in front of a plate (along the plane of symmetry $y = 0$) at $M_\infty = 2$, $m = 1$, and $d_\infty = 200$ μm. The origin of the x axis corresponds to the position of the front surface of the plate. The dashed lines correspond to flow about the plate of a gas not containing drops, while the solid and dot-dash lines correspond to flow of a gas with drops in the presence and absence of drop fragmentation and vaporization, respectively (the dotted line shows the corrected density of the drops). The thin solid lines show the erosion of the shock front by artificial viscosity. It is evident that in the absence of stripping and vaporization of drops, the density of the vapor in the shock layer behaves nonmonotonically; it increases immediately after the shock and then decreases in front of the plate. This pattern is connected with the fact that drops behind the shock front are greatly slowed. Here, as a result of the work done by the phase interaction force, some of the kinetic energy of the drops is converted to the thermal energy of the vapor. The temperature and pressure of the vapor in the shock layer ahead of the body increase dramatically. Meanwhile, the temperature increase outstrips the pressure increase, so that the density ahead of the plate decreases. In the absence of drop fragmentation and vaporization, the corrected density of the drops in the shock layer increases monotonically due to their deceleration.

The presence of stripping of the drops intensifies the decay of the shock wave and significantly increases the density of the vapor in the wave due to the additional injection of vapor during vaporization of the separated fine drops; the corrected density of the coarse drops decreases monotonically, which is connected with intensive stripping of their surface layer. At the above-indicated values of the governing parameters, the drops are nearly completely stripped and vaporized at the distance $\bar{x} \approx 1.6$. Thus, they do not reach the surface of the plate.

Figure 2 shows the characteristic form of the dependence of the decay of the shock wave $\bar{x}_s = x_s/h$ (along the plane of symmetry $y = 0$) on the initial diameter of the drops d_∞ (line 2) and their relative mass content m (line 1) in the presence (solid lines) and absence (dot-dash lines) of stripping and vaporization [the relation $\bar{x}_s(d_\infty)$ was obtained at $m = 1$, while $x_s(m)$ was obtained at $d_\infty = 400$ μm; $M_\infty = 2$ for both cases]. It is evident that the behavior of the dependence of $\bar{x}_s$ on d_∞ is qualitatively different: in the presence of stripping, $\bar{x}_s$ decreases monotonically with an increase in d_∞. Conversely, in the absence of stripping, it increases monotonically (the value of $\bar{x}_s$ is the same in both cases in the limit at $d_\infty \to \infty$). Here, stripping has a strong effect on decay of the wave for sufficiently fine drops ($d_\infty \lesssim 400$ µm), while this effect is less pronounced for coarse drops ($d_\infty \gtrsim 600$ µm). The vaporization of coarse drops has the greatest effect on the decay of the shock wave in the limiting case $d_\infty = 0$.

Similarly to the relation $\bar{x}_s(d_\infty)$, the behavior of $\bar{x}_s(m)$ is qualitatively different in the presence and absence of drop stripping. An increase in m is accompanied by an increase in the intensity of mechanical phase interaction. In the absence of stripping, an increase in m reinforces the phenomenon of "pulling" of the shock front toward the body by the disperse phase [1]. In the presence of stripping and vaporization, the rate of vapor injection increases with an increase in m. Meanwhile, the effect of additional injection of vapor is stronger than the "pulling" effect, so the decay of the shock wave increases with an increase in m.

Figure 3 shows the characteristic form of the dependence of the decay of the shock wave $\bar{x}_s$ on M_∞ in the presence (solid lines) and absence (dot-dash lines) of drop fragmentation and vaporization ($m = 1$, $d_\infty = 400$ µm), as well as the results obtained for the case of flow about a plate of a gas without drops (dashed lines). It is interesting to note that the effect of stripping and vaporization on decay of the shock weakens with an increase in M_∞. In the absence of fragmentation and vaporization, the presence of drops in the flow leads to a reduction in decay of the wave throughout the investigated range of M_∞. Stripping and subsequent vaporization of separated fine drops usually lead to a marked increase in $\bar{x}_s$.

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